

Hong Kong Mathematics Olympiad (2009 / 2010)

Final Event 1 (Individual)

香港数学竞赛 (2009 / 2010)

决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 把三个体积分别为 1, 8, 27 的正立方体，以面贴面的方法黏合起来。若 a 为所得的多面体的最小总表面积，求 a 的值。

Three cubes with volumes 1, 8, 27 are glued together at their faces. If a is the smallest possible surface area of the resulting polyhedron, find the value of a .

2. 已知 $f(x) = -x^2 + 10x + 9$ ，且 $2 \leq x \leq \frac{a}{9}$ 。若 b 是 f 的最大及最小值之差，求 b 的值。

Given that $f(x) = -x^2 + 10x + 9$, and $2 \leq x \leq \frac{a}{9}$. If b is the difference of the maximum and minimum values of f , find the value of b .

3. 已知 p 及 q 是整数，且 $pq = b$ 及 $p^2q + q^2p + p + q = 70$ 。若 $c = p^2 + q^2$ ，求 c 的值。

Given that p and q are integers with $pq = b$ and $p^2q + q^2p + p + q = 70$. If $c = p^2 + q^2$, find the value of c .

4. 在一个有 c 行的演奏厅中，每一行都比前一行多两个座位。若中间的行有 64 个座位，这演奏厅共有多少个座位？

There are c rows in a concert hall and each succeeding row has two more seats than the previous row. If the middle row has 64 seats, how many seats does the concert hall have?

Hong Kong Mathematics Olympiad (2009 / 2010)

Final Event 2 (Individual)

香港数学竞赛 (2009 / 2010)

决赛项目 2 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 a, p, q 是质数，且满足 $a < p$ 及 $a + p = q$ ，求 a 的值。

If a, p, q are primes with $a < p$ and $a + p = q$, find the value of a .

2. 若 b 及 h 为正整数，且满足 $b < h$ 及 $b^2 + h^2 = b(a + h) + ah$ ，求 b 的值。

If b and h are positive integers with $b < h$ and $b^2 + h^2 = b(a + h) + ah$, find the value of b .

3. 在一个 $(2b + 1) \times (2b + 1)$ 的棋盘上任意选取两个不在同一横行上方格。若 c 为选取的两个不同方格的组合数目，求 c 的值。

In a $(2b + 1) \times (2b + 1)$ checkerboard, two squares not lying in the same row are randomly chosen.

If c is the number of combinations of different pairs of squares chosen, find the value of c .

4. 已知 $f(x) = c \left| \frac{1}{x} - \left\lfloor \frac{1}{x} + \frac{1}{2} \right\rfloor \right|$ ，其中 $\lfloor x \rfloor$ 是小于或等于实数 x 的最大整数。若 d 为 $f(x)$ 的最大值，求 d 的值。

Given that $f(x) = c \left| \frac{1}{x} - \left\lfloor \frac{1}{x} + \frac{1}{2} \right\rfloor \right|$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to the real number x . If d is the maximum value of $f(x)$, find the value of d .

Hong Kong Mathematics Olympiad (2009 / 2010)

Final Event 3 (Individual)

香港数学竞赛 (2009 / 2010)

决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 a 为 15147 的相异质因子的数目。求 a 的值。

If a is the number of distinct prime factors of 15147, find the value of a .

2. 若 $x + \frac{1}{x} = a$ 及 $x^3 + \frac{1}{x^3} = b$, 求 b 的值。

If $x + \frac{1}{x} = a$ and $x^3 + \frac{1}{x^3} = b$, find the value of b .

3. 设 $f(x) = \begin{cases} x+5 & \text{当 } x \text{ 是一奇数} \\ \frac{x}{2} & \text{当 } x \text{ 是一偶数} \end{cases}$ 。若 c 是一奇数及 $f(f(f(c))) = b$, 求 c 的最小值。

Let $f(x) = \begin{cases} x+5 & \text{when } x \text{ is an odd integer} \\ \frac{x}{2} & \text{when } x \text{ is an even integer} \end{cases}$. If c is an odd integer and $f(f(f(c))) = b$, find the least value of c .

4. 设 $f\left(\frac{x}{3}\right) = x^2 + x + 1$ 。若 d 为所有满足 $f(3x) = c$ 的 x 之和，求 d 的值。

Let $f\left(\frac{x}{3}\right) = x^2 + x + 1$. If d is the sum of all x for which $f(3x) = c$, find the value of d .

Hong Kong Mathematics Olympiad (2009 / 2010)

Final Event 4 (Individual)

香港数学竞赛 (2009 / 2010)

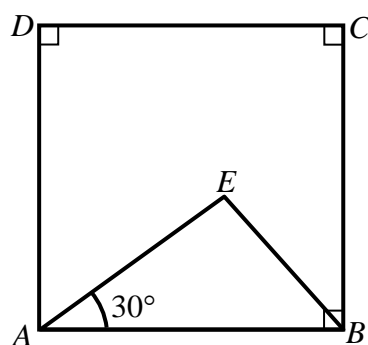
决赛项目 4 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 在图一中， $ABCD$ 为一正方形， E 为此正方形以外的一点及 $\angle EAB = 30^\circ$ 。若 $ABCD$ 的面积是 ABE 的面积的六倍，则 $AE : AB = a : 1$ 。求 a 的值。

In Figure 1, $ABCD$ is a square, E is a point outside of the square and $\angle EAB = 30^\circ$. If the area of $ABCD$ is six times that of ABE , then the ratio $AE : AB = a : 1$. Find the value of a .



图一

Figure 1



2. 已知 $b = \frac{\log 8^a + \log 27^a + \log 125^a}{\log 9 + \log 25 + \log 2 - \log 15}$ ，求 b 的值。

Given that $b = \frac{\log 8^a + \log 27^a + \log 125^a}{\log 9 + \log 25 + \log 2 - \log 15}$, find the value of b .



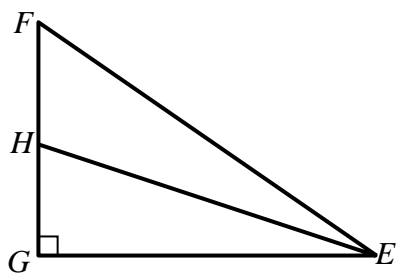
3. 设 c 为 $1^3 + 2^3 + \cdots + 2009^3 + 2010^3$ 除以 b^2 的余数。求 c 的值。

Let c be the remainder of $1^3 + 2^3 + \cdots + 2009^3 + 2010^3$ divided by b^2 . Find the value of c .



4. 在图二中， EFG 为一直角三角形。已知 H 为 FG 上的一点，使得 $GH:HF=4:5$ 及 $\angle GEH = \angle FEH$ 。若 $EG=c$ 及 $FG=d$ ，求 d 的值。

In Figure 2, EFG is a right-angled triangle. Given that H is a point on FG , such that $GH:HF=4:5$ and $\angle GEH = \angle FEH$. If $EG=c$ and $FG=d$, find the value of d .



图二
Figure 2

